Midterm Exam
Electrodynamics
Winter, '05

Choose one of the following two problems. If you hand in solutions to both problems and do not indicate which problem is to be graded, you will get the worse score of the two. Good luck.

Problem 1: A very long hollow square cross-section tube has three of its four walls grounded. The length of each side of the square is a. The fourth wall has the following voltage distribution:

$$V = \begin{cases} 2V_0 \dfrac{x}{a} & \text{if} \quad 0 \le x < \dfrac{a}{2} \\ 2V_0\left(1 - \dfrac{x}{a}\right) & \text{if} \quad \dfrac{a}{2} \le x \le a \end{cases}.$$

Find the electrostatic potential everywhere inside the tube and the charge density on the wall opposite the wall with the varying voltage.

Problem 2: A charge of $+Q$ is placed on the end of a small rod of length a and a charge of $-Q$ is placed on the other end. The rod is then brought to a distance $d \gg a$ from a grounded conducting plane. Find the charge density on the plane to lowest order in a/d as a function of angle θ, the angle between the rod and the normal of the plane.

Possibly useful integrals:

$$\int x \sin \alpha x dx = \frac{\sin \alpha x - \alpha x \cos \alpha x}{\alpha^2} \qquad \int x^2 \sin \alpha x dx = \frac{2\alpha x \sin \alpha x - \left(\alpha^2 x^2 - 2\right)\cos \alpha x}{\alpha^3}$$

$$\int x \cos \alpha x dx = \frac{\cos \alpha x + \alpha x \sin \alpha x}{\alpha^2} \qquad \int x^2 \cos \alpha x dx = \frac{2\alpha x \cos \alpha x + \left(\alpha^2 x^2 - 2\right)\sin \alpha x}{\alpha^3}$$