Final Exam
Electrodynamics
March 16, 2005

Do one and only one problem in each of the following two sections. If you hand solutions or partial solutions to more than one problem, clearly indicate which problem is to be graded. If you hand in more than one problem to a section and do not indicate which problem is to be graded, the lesser of the two grades will be recorded.

Section I

Problem 1: A long superconducting rod of radius R is placed in a uniform magnetic field of strength H_0 perpendicular to the rod. A true superconductor is a perfect diamagnetic material with $\mu = 0$. Begin this problem by finding the magnetic field, magnetic induction and magnetization everywhere both inside the rod and outside the rod for a μ finite, but close to zero and then find the limit of these quantities for the case when $\mu \to 0$. We often say that superconductors exclude magnetic fields. If we are being precise, are we talking about the field or the induction?

Problem 2: An electric dipole is situated at the center of a spherical cavity in a conductor. The cavity has radius R and the conductor is grounded. Immediately around the dipole is vacuum, but at a dielectric with dielectric constant ε begins at radius of $R/2$ and continues out to the conductor. Find the electric field and the electric displacement everywhere. Find the polarization in the dielectric. What is $\vec{\nabla} \bullet \vec{P}$ in the dielectric?

Section II

Problem 1: Find the Green's function for the inside of a sphere subject to Dirichlet boundary conditions on its surface.

Problem 2: A hard iron sphere of radius R is magnetized with the following magnetization function:

$$\vec{M}(r,\theta,\phi) = \frac{r^2}{2}\left(5\cos^3\theta - 3\cos\theta\right)\hat{r} - \frac{r^2}{2}\sin\theta\left(5\cos^2\theta - 1\right)\hat{\theta}$$
$$= r^2 P_3^0(\cos\theta)\hat{r} - \frac{r^2}{3}P_3^1(\cos\theta)\hat{\theta}$$

(Incidentally, $\vec{\nabla} \bullet \vec{M}(r,\theta,\phi) = 0$ inside the sphere.) Find the magnetic induction everywhere both inside and outside the sphere.

See next page for possibly helpful equations.

Cylindrical Coordinates:

$$\vec{\nabla}\Psi = \frac{\partial \Psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \Psi}{\partial \theta}\hat{\theta} + \frac{\partial \Psi}{\partial z}\hat{z}$$

$$\vec{\nabla}\bullet\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

Spherical Coordinates:

$$\vec{\nabla}\Psi = \frac{\partial \Psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \Psi}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial \Psi}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\bullet\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Other relationships:

$$\frac{1}{\left|x - x'\right|} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}^*\left(\theta',\phi'\right)Y_{lm}\left(\theta,\phi\right)$$

$$Y_{lm}\left(\theta,\phi\right) = \sqrt{\frac{2l+1}{4\pi}\frac{\left(l-m\right)!}{\left(l+m\right)!}}P_l^m\left(\cos\theta\right)e^{im\phi}$$

$$P_l^m\left(z\right) = \left(-1\right)^m\left(1-z^2\right)^{m/2}\frac{d^m}{dz^m}P_l\left(z\right)$$

If you need other relationships, ask!