Electrodynamics
Spring, '05
Problem Set 6
Double Problem Set

Due: At final, June 6, 2005

Problem 1: Jackson 11.13

Problem 2: Jackson 11.13 except consider the wire to have a uniform current *I* in the inertial frame *K'* and no charge density. Explain why a charge density can appear on the wire even though no charge was initially present. Why doesn't this violate conservation of charge?

Problem 3: Jackson 11.23

Problem 4: A massless muon-type neutrino can interact with either a neutron (mass = 940 MeV/c^2) to give a muon (mass 105 MeV/c^2) and a proton (mass = 939 MeV/c^2) or an electron (mass .511 MeV/c^2), a muon, and an electron neutrino. If the incoming neutrinos are 100 GeV, find the maximum scattering angle that the muon can come out at in the lab.

Problem 5: A π^0 decays into two photons. If the π^0 has momentum $\vec{p}$ in the lab and if the decay of the π^0 is independent of angle in its rest frame, find:

a) the probability density distribution of the photon's laboratory energy;
b) the angle between the two photons in the laboratory as a function of the decay angle in the center of mass (the angle between the direction of the pion transformation to the lab and the direction of one of the two photons; and
c) the minimum of this angle.

Problem 6: Consider a system with two massive, real valued scalar fields, ϕ_1 and ϕ_2. Assume that both fields have the same mass m_0.

a) Write down a Lagrangian density that will give the Klein-Gordon equation as the equation of motion for each of these fields.
b) Show that your Lagrangian is invariant under the transformation (θ is independent of position):
$$\phi_1 \to \cos\theta\phi_1' + \sin\theta\phi_2'$$
$$\phi_2 \to -\sin\theta\phi_1' + \cos\theta\phi_2'$$
c) Is the Lagrangian invariant under this transformation if $m_1 \neq m_2$?
d) If $\theta = \theta(\vec{x})$, the Lagrangian is no longer invariant. A gauge field must be added to restore invariance. Find this gauge field that makes the Lagrangian

invariant. Find the kinetic energy term for this gauge field. Write down the entire Lagrangian for both the scalar fields and the gauge field.

e) Find the equations of motion for ϕ_1, ϕ_2, and the gauge field from the Lagrangian given in the previous part.
f) Derive the stress-energy tensor for this Lagrangian.

Problem 7: Jackson 12.16

Problem 8: A particle of mass m and velocity $v_z\hat{z}$ travels through a crossed electric and magnetic field. The electric field is in the $\hat{x}$ direction and the magnetic field is in the $\hat{y}$ direction. Find the ratio of the magnitude of these two fields such that the particle's trajectory is not deflected. If the fields have this ratio in the lab frame, show that the particle measures no field in its rest frame.

Problem 9: We started this unit on special relativity by showing that if two oppositely charged particles were moving with a velocity $\vec{v}$ in the laboratory frame, we would measure a force between them of $F_{lab} = \frac{dp_{\perp,lab}}{dt_{lab}} = -\frac{q^2}{4\pi\varepsilon_0 r^2}\left(1-\frac{v^2}{c^2}\right)$. In the particles' rest frame, the force is $F_{cm} = \frac{dp_{\perp,cm}}{dt_{cm}} = -\frac{q^2}{4\pi\varepsilon_0 r^2}$. Show that these two are consistent with the Lorentz transformations that we have developed. (Remember that force is not a 4-vector.)

Qualifying Exam Problems

Spring '90 – Problem 1

An infinite sheet of current flows in the $+x$ direction in the $z=0$ plane. The current density is j amps/meter.

a) Find the magnetic field generated by this current everywhere.
b) Find the electric and magnetic fields observed by a person traveling at a velocity V toward the plane from the $+z$ direction.

Fall '94 – Problem 5

A radiation field is represented by the vector potential

$$\vec{A} = A_0 e^{i\phi(x,y,z)}\hat{y}$$

where $\phi(x,y,z) = k_x x + k_y y - \omega t$. For this potential, find:

a) The direction and magnitude of the energy flow of the radiation;

b) The scalar potential that would make the representation of the potentials that of the Lorentz gauge, $\left(\vec{\nabla}\bullet\vec{A}+\frac{1}{c^2}\frac{\partial\Phi}{\partial t}=0\right)$; and

c) The gauge transformation that would transform vector potential and the scalar potential from part (b) into the radiation gauge $\left(\vec{\nabla}\bullet\vec{A}=0\right)$.