Problem Set 4
Electrodynamics
Due: Feb. 4, 2005

Problem 1: The linear transform

$$\left(\frac{z-z_1}{z-z_2}\right)/\left(\frac{z_3-z_1}{z_3-z_2}\right)=\left(\frac{w-w_1}{w-w_2}\right)/\left(\frac{w_3-w_1}{w_3-w_2}\right)$$

is a conformal mapping which maps the points $\{z_1,z_2,z_3\}$ to $\{w_1,w_2,w_3\}$ in such a way as to transform circles and straight lines into circles and straight lines. Use this to transform the potential between two concentric cylinders to that for two non-concentric cylinders. Specifically, find the potential everywhere between two cylinders where one is inside the other and offset from the center by .2 cm. The inner cylinder has a radius of .1 cm and is grounded and the outer cylinder has a radius of 1 cm and is held at 100 V. Plot the potential and field lines. Hint: Choose $\{z_1,z_2,z_3\}=\{1,0,-1\}$ and $\{w_1,w_2,w_3\}=\{1,\mu,-1\}$ where μ and the radius of the concentric cylinder in the z plane are chosen such that the inner cylinder radius and center fall in the correct place.

Problem 2: The linear transform mentioned in problem 1 can also transform concentric cylinders into side by side cylinders. Find the transform that two concentric cylinders into two side by side cylinders and evaluate the potential for two 1 mm wires separated by 3 mm. Does the result make sense in terms of image charges?

Problem 3: A rectangular tube with 2 cm sides and 1 cm top and bottom is charged such that the left and top edges are held at 100 volts and the bottom and right edges are grounded. Use the Schwartz-Christoffel transform to calculate the potential at points inside the tube. Plot the potential (use AspectRatio->1 to get a realistic looking plot). Show that going into the corners where both adjacent sides are at the same potential varies as ρ, the distance from the corner while going into the other corner, the field varies as $1/\rho$. Please feel free to use my notebook at the basis for your solution modifying it as needed.

Problem 4: Use the expansion in orthogonal polynomials to solve problem 3. Can you demonstrate the same behavior in the corners?

Problem 5: Use orthogonal polynomials to solve this problem. Find the potential inside a cylinder whose surface is held at the following potential:

$$V(R,\theta)=V_0\sin^2\theta.$$

Plot the potential in the interior of the cylinder. Also plot the potential at the surface of the cylinder for your result to make sure that you included all necessary term.

Short Answer Problems from Qualifying Exams

Do not use *Mathematica* for these problem!

Spring '96

I. Consider an object of arbitrary shape, composition, and charge distribution suspended a distance x above a uniformly charged infinite plane. It experiences a force F due to the charges on the plane. If it is moved to a distance $2x$, what force does it now experience? Why?

II. A point charge q is located a distance b from the center of a grounded conducting sphere of radius a. Does the electrostatic force between the charge and the sphere pull these two objects together or push them apart? Why? Now suppose the conducting sphere is not grounded and carries no net charge. Now does the electrostatic force pull the two objects together or push them apart? Why? Is the magnitude of the force greater or less than the force between the charge and grounded sphere? Why

Fall, '95

III. A charge Q is brought to a distance d of a grounded infinite conducting plane. Find the charge density on the surface of the plane.