Problem Set 2
Electrodynamics
Due: Jan. 21, 2005

Problem 1: In *Mathematica*, define a function that does the line integral of a vector function $\vec{V} = \{V_x, V_y\}$ around a circle of radius r centered at $\{x, y\}$ in the xy plane:

$$\oint \vec{V} \bullet d\vec{\ell}.$$

Use this function to evaluate the line integral of the vector function:

$$\vec{V} = \frac{-y}{x^2+y^2}\hat{x} + \frac{x}{x^2+y^2}\hat{y}$$

around the circle of radius 1 centered at the origin and of radius .5 centered at {1,0}. Compare your answers with those obtained in the first problem set.

Problem 2: Modify the function you wrote in problem 1 to do integrals in the complex plane of complex functions where you can explicitly write out the real and imaginary parts. Integrate the functions $1/\left(z-(1+i)\right)$ and $z/\left(z-(1+i)\right)^2$ around a circle centered at $(1+i)$ with radii 1 and 2 and show that you get what you expect from Cauchy's theorem.

Problem 3: In *Mathematica*, the Bessel functions of the first kind ($J_\nu(x)$)of order ν are represented by the function **BesselJ**[ν,x]. Using Mathematica, plot the order 3 Bessel function. As you can see from the plot, the function oscillates. At large x, it can be approximated by

$$J_\nu(x) \approx \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) \qquad \text{Jackson 3.91}$$

Plot both the Bessel function and the approximation out to x=50 and see how well they come together. Like the sines and cosines in a Fourier expansion, the Bessel functions of order ν can be used to approximate an arbitrary function over a finite interval L. This expansion is called the Bessel-Fourier expansion and is written as

$$f(x) = \sum_{n=1}^{\infty} A_{\nu,n} J_\nu\left(\frac{z_{\nu,n}x}{L}\right) \qquad \text{Jackson 3.96}$$

where $z_{\nu,n}$ is the n^{th} zero of the ν order Bessel function and

$$A_{\nu,n} = \frac{2}{L^2 J_{\nu+1}^2\left(z_{\nu,n}\right)}\int_0^L f(x) J_\nu\left(\frac{z_{\nu,n}x}{L}\right) x\,dx. \qquad \text{Jackson 3.97}$$

Find the first 5 zeros (use **FindRoot**) of the ν=3 function. Evaluate the first 5 $A_{3,n}$ for the function

$$f(x) = \sin\left(\frac{\pi x}{L}\right).$$

Plot the sum of the first 5 terms and compare it to the function itself.

Problem 4: Use the functions in the *Mathematica* package **VectorAnalysis** to find the form of the potential

$$\Phi = \frac{1}{|\vec{r} - \vec{r}_1|} + \frac{1}{|\vec{r} - \vec{r}_2|}$$

where $r_1 = \{0, 0, d\}$ and $r_2 = \{0, 0, -d\}$ in Prolate Spheroidal coordinates (as in last week's problem set). Take the gradient in these coordinates to field the field. Show that the curl and divergence are zero in these coordinates for all points except possibly at r_1 and r_2.

Spring '96 Dynamics and Math Physics Qualifying Exam

Do not use *Mathematica* for this problem!

Show that the following differential equations can be solved using the Green's function method.

(a) $\frac{d^2u}{dx^2} = f(x)$ with the boundary conditions $u(0) = u(L) = 0$; and

(b) $\frac{d^2y}{dx^2} + k^2 y = g(x)$ with the boundary conditions $y(0) = y(L) = 0$ and $k \neq n\pi / L$ for any integer n.

Show that your solution to (b) could be utilized to solve the problem of an externally driven string. The relevant partial differential equation is $T\frac{\partial^2 u}{\partial x^2} - \rho\frac{\partial^2 u}{\partial t^2} = F(x,t)$. Assume that the time dependence of the external force is sinusoidal $F(x,t) = f(x)e^{i\omega t}$.