Electrodynamics
Spring, '05
Problem Set 2

Qualifying Exam Problem 2

The system is cylindrically symmetric around the z axis. Thus the solution for the potential must be of the form:

$$\Phi = \sum \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l\left(\cos\theta\right)$$

Since there are only P_3 and P_5 in the problem, the solution should look like:

$$\Phi = \left(A_3 r^3 + \frac{B_3}{r^4} \right) P_3\left(\cos\theta\right) + \left(A_5 r^5 + \frac{B_5}{r^6} \right) P_5\left(\cos\theta\right)$$

At R_a, the coefficient of P_3 must be V_a and the coefficient of P_5 must be zero. Likewise at R_b, the coefficient of P_5 must be V_b and the coefficient of P_3 must be zero. This gives 4 equations:

$$A_3 R_a^3 + \frac{B_3}{R_a^4} = V_a$$

$$A_3 R_b^3 + \frac{B_3}{R_b^4} = 0$$

$$A_5 R_a^5 + \frac{B_5}{R_a^6} = 0$$

$$A_5 R_b^5 + \frac{B_5}{R_b^6} = V_b$$

The solution of these equations is

$$A_3 = -\frac{R_a^4 V_a}{R_b^7 - R_a^7}$$

$$B_3 = \frac{R_b^7 R_a^4 V_a}{R_b^7 - R_a^7}$$

$$A_5 = \frac{R_b^6 V_b}{R_b^{11} - R_a^{11}}$$

$$B_5 = -\frac{R_a^{11} R_b^6 V_b}{R_b^{11} - R_a^{11}}$$

Giving a potential:

$$\Phi = \frac{R_a^4 V_a}{R_b^7 - R_a^7}\left(-r^3 + \frac{R_b^7}{r^4}\right)P_3\left(\cos\theta\right) + \frac{R_b^6 V_b}{R_b^{11} - R_a^{11}}\left(r^5 - \frac{R_a^{11}}{r^6}\right)P_5\left(\cos\theta\right)$$