Electrodynamics
Spring, '05
Problem Set 2

Problem 2
Jackson 6.17

Part (a)

The Lorentz force on a charged particle is:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

If all particle have a constant ratio of electric to magnetic charge, then the duality transform of $\vec{E}' = \vec{E}\cos\xi + cB\sin\xi$ and $\vec{B}' = \vec{B}\cos\xi - \frac{\vec{E}}{c}\sin\xi$ will change Maxwell's equations with magnetic charges to the standard form. Using this in the Lorentz force gives:

$$\vec{F} = q\left(\vec{E}' + \vec{v} \times \vec{B}'\right)$$
$$= q\cos\xi\vec{E} + q\sin\xi c\vec{B} + q\cos\xi\vec{v} \times \vec{B} - \frac{q\sin\xi}{c} v \times E$$

At this point, $q_e = q\cos\xi$ and to get Jackson's equation, $q_m = qc\mu_0\sin\xi$. This gives a constant ratio of $q_m / q_e = \sqrt{\mu_0 / \varepsilon_0}\tan\xi = Z_0\tan\xi$. This gives the Lorentz force of

$$\vec{F} = q_e\vec{E} + q_m\frac{\vec{B}}{\mu_0} + q_e\vec{v} \times \vec{B} - q_m\varepsilon_0\vec{v} \times \vec{E}$$

Part (b)

For general charges, you have to transform the charges as well as the fields. First transform the fields:

$$\vec{F} = q_e'\vec{E}' + q_m'\frac{\vec{B}'}{\mu_0} + q_e'\vec{v}\times\vec{B}' - q_m'\varepsilon_0\vec{v}\times\vec{E}'$$

$$= q_e'\left(\vec{E}\cos\xi + c\vec{B}\sin\xi\right) + q_m'\frac{\left(\vec{B}\cos\xi - \frac{\vec{E}}{c}\sin\xi\right)}{\mu_0} +$$

$$q_e'\vec{v}\times\left(\vec{B}\cos\xi - \frac{\vec{E}}{c}\sin\xi\right) - q_m'\varepsilon_0\vec{v}\times\left(\vec{E}\cos\xi + c\vec{B}\sin\xi\right)$$

$$= \left(q_e'\cos\xi - \frac{q_m'\sin\xi}{\mu_0 c}\right)\vec{E} + \left(q_m'\cos\xi + q_e'\mu_0 c\sin\xi\right)\frac{\vec{B}}{\mu_0} +$$

$$\left(q_e'\cos\xi - \frac{q_m'\sin\xi}{\mu_0 c}\right)\vec{v}\times\vec{B} - \left(q_m'\cos\xi + q_e'\mu_0 c\sin\xi\right)\varepsilon_0\vec{v}\times\vec{E}$$

Thus, if $q_e = q_e'\cos\xi - q_m'\sin\xi / Z_0$ and $q_m = q_m'\cos\xi + q_e'Z_0\sin\xi$, then the force is invariant.

Part (c)

Equation 6.155 gives you the impulse on particle 2 with electric charge due to a magnetic charge of at the origin. If there were an additional electric charge at the origin and an additional magnetic charge at 2, you would get $\Delta p_y = \frac{e_2 g_1 - e_1 g_2}{2\pi b}$. The minus sign comes from the sign difference in the cross product terms for the electric and magnetic fields. The angular momentum changes is $\Delta L_z = \frac{e_2 g_1 - e_1 g_2}{2\pi}$ and if this is quantized in units of $\hbar$, this becomes $e_2 g_1 - e_1 g_2 = nh$. Under a duality transform (the same ξ applies to all particles),

$$e_2 g_1 - e_1 g_2 = \left(e_2'\cos\xi - \frac{g_2'\sin\xi}{Z_0}\right)\left(g_1'\cos\xi + e_1'Z_0\sin\xi\right) - \left(e_1'\cos\xi - \frac{g_1'\sin\xi}{Z_0}\right)\left(g_2'\cos\xi + e_2'Z_0\sin\xi\right)$$

$$= e_2'g_1'\cos^2\xi - g_2'e_1'\sin^2\xi + e_2'g_1'\sin^2\xi - g_2'e_1\cos^2\xi$$

$$= e_2'g_1' - g_2'e_1'$$