Electrodynamics
Problem Set 2
Spring, '05

Problem 1
Jackson 6.15

Part (a)

In order for the form of the relationship to be invariant under special rotations, both sides of the equation must transform as vectors. Thus terms proportional to $\left|\vec{J}\right|^2$ or $\left|\vec{H}\right|^2$ are not allowed. Possible vector combinations up to second order in $\vec{J}$ and $\vec{H}$ are $\vec{J}$, $\vec{H}$, $\vec{H}\times\vec{J}$, $\left|\vec{H}\right|^2\vec{J}$, $\left(\vec{H}\bullet\vec{J}\right)\vec{H}$, $\left(\vec{H}\bullet\vec{J}\right)\vec{J}$, $\left|\vec{J}\right|^2\vec{H}$, $\left(\vec{H}\bullet\vec{J}\right)\vec{H}\times\vec{J}$. Under a parity transformation these terms are odd, even, odd, odd, odd, even, even, and even respectively. Since $\vec{E}$ is odd, only the first, third, fourth, and fifth terms have the same parity giving the equation in part (a) of this problem.

Part (b)

Under time reversal, of the remaining terms, the first term is odd, the second term is even, and the third and fourth terms are odd. The electric field is even. Thus, if time reversal were a good symmetry for this system, only the second term would remain. We know that the first term represents Joule heating and is a dissipative terms which removes energy from the system. Dissipative terms are not time reversal invariant. Thus the third can be present as a magnetic induced resistance. The fourth term generally is zero for Hall type experiments because $\vec{H}\perp\vec{J}$. In general, this would represent work done by the magnetic field. Since magnetic fields do no work, this term must be zero.