Electrodynamics
Problem Set 1
Spring, '05

Problem 2
Jackson 6.1

Part (a)

Starting with equation 6.45 and 6.44 we have

$$\Psi(x,y,t)=\int\frac{\delta\left(t'-\left(t-\frac{|x-x'|}{c}\right)\right)}{|x-x'|}\delta(x')\delta(y')\delta(t')\,dx'dy'dz'dt'$$

$$=\int\frac{\delta\left(\frac{\sqrt{\rho^2+(z-z')^2}}{c}-t\right)}{\sqrt{\rho^2+(z-z')^2}}dz'$$

$$=\begin{cases}0 & if \quad \rho>ct\\ \left.\dfrac{1}{\sqrt{\rho^2+(z-z')^2}}\dfrac{1}{\left|\dfrac{\partial}{\partial z'}\dfrac{\sqrt{\rho^2+(z-z')^2}}{c}\right|}\right|_{z'=z\pm\sqrt{c^2t^2-\rho^2}} & if \quad \rho<ct\end{cases}$$

$$\Psi(x,y,t)=\left(\left.\frac{1}{\sqrt{\rho^2+(z-z')^2}}\frac{1}{\left|\frac{\partial}{\partial z'}\frac{\sqrt{\rho^2+(z-z')^2}}{c}\right|}\right|_{z'=z-\sqrt{c^2t^2-\rho^2}}\right)\Theta(ct-\rho)$$

$$\Psi(x,y,t)=\left(\left.\frac{1}{\sqrt{\rho^2+(z-z')^2}}\frac{c\sqrt{\rho^2+(z-z')^2}}{2\frac{1}{2}|(z-z')|}\right|_{z'=z\pm\sqrt{c^2t^2-\rho^2}}\right)\Theta(ct-\rho)$$

$$\Psi(x,y,t) = \left.\frac{c\Theta(ct-\rho)}{\left|(z-z')\right|}\right|_{z'=z\pm\sqrt{c^2t^2-\rho^2}}$$

$$= \frac{c\Theta(ct-\rho)}{\left|\sqrt{c^2t^2-\rho^2}\right|} + \frac{c\Theta(ct-\rho)}{\left|-\sqrt{c^2t^2-\rho^2}\right|} = \frac{2c\Theta(ct-\rho)}{\sqrt{c^2t^2-\rho^2}}$$

Since you are pulsing the wire everywhere simultaneously, the pulse from far pieces of the wire comes later. Hence the theta function. The pieces of the wire that are arriving at later times are like point sources and therefore the amplitude falls off as 1/z .

Part (B)

For this part, $f(\vec{x}',t') = \delta(x')\delta(t')$ giving

$$\Psi(x,y,t) = \int \frac{\delta\left(t' - \left(t - \frac{|x-x'|}{c}\right)\right)}{|x-x'|}\delta(x')\delta(t')\,dx'dy'dz'dt'$$

$$= \int \frac{\delta\left(\frac{\sqrt{x^2+(y-y')^2+(z-z')^2}}{c} - t\right)}{\sqrt{x^2+(y-y')^2+(z-z')^2}}dy'dz'$$

$$= \int \frac{\delta\left(\frac{\sqrt{x^2+\rho^2}}{c} - t\right)}{\sqrt{x^2+\rho^2}}\rho\, d\rho\, d\theta = 2\pi\int \frac{\delta\left(\frac{\sqrt{x^2+\rho^2}}{c} - t\right)}{\sqrt{x^2+\rho^2}}\rho\, d\rho$$

The integral is non-zero only if $ct > |x|$ (since the minimum $\rho > 0$) resulting in the theta function.

$$\Psi(x,y,t) = 2\pi\int \frac{\delta\left(\frac{\sqrt{x^2+\rho^2}}{c} - t\right)}{\sqrt{x^2+\rho^2}}\rho\, d\rho$$

$$= \left.\frac{2\pi\Theta(ct-|x|)}{\sqrt{x^2+\rho^2}}\frac{c\sqrt{x^2+\rho^2}}{\rho}\rho\right|_{\rho=\sqrt{c^2t^2-x^2}}$$

$$= 2\pi c\Theta(ct-|x|)$$

In this case, the number of radiating points increase with ρ and therefore the function does not have the $1/\rho$ in it like the line of charge does.