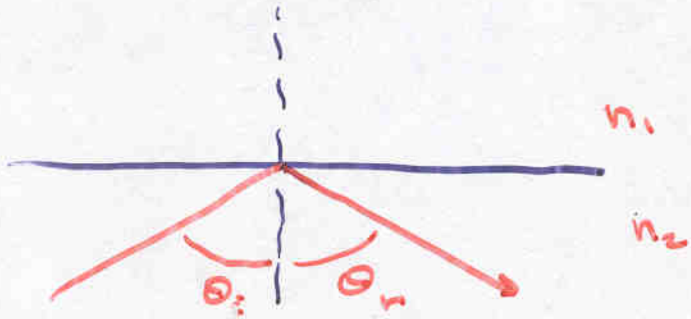
Total Internal Reflection



$n_2 > n_1$

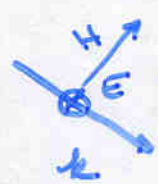
$$\sin\theta_t = \frac{n_2}{n_1}\sin\theta_i \qquad \text{Snell's law}$$

$$\geq 1$$

At = 1 Energy transmitted along surface, does not escape

⇒ all energy is reflected

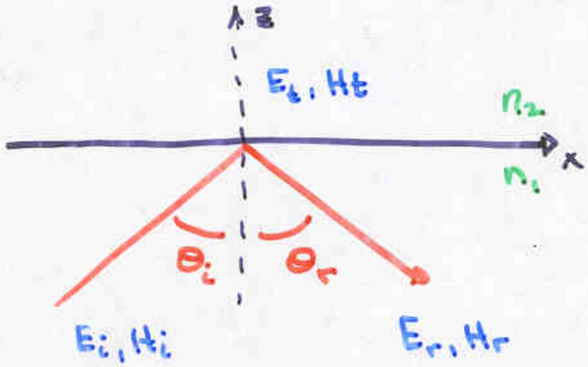
<u>Problem:</u>



$E_T = 0$ ⇒ continuous

$H_T \neq 0$ ⇒ problem.

Total Internal Reflection (Carson & Lorrain)



Condition for total internal reflection

$$\sin\theta_i > \frac{n_2}{n_1}$$

Electric Field Amplitudes

$$\vec{E}_i = \vec{E}_{io}\, e^{i(\vec{k}_i\cdot x - \omega_i t)}$$

$$\vec{E}_r = \vec{E}_{or}\, e^{i(k_r^x x + k_r^z z - \omega_r t)}$$

$$\vec{E}_t = \vec{E}_{ot}\, e^{i(k_t^x x + k_t^z z - \omega_t t)}$$

Kinematic Conditions (phase conditions at interface)

time invariance: $\omega_i = \omega_r = \omega_t = \omega$

x - invariance: $k_i^x = k_i \sin\theta_i = k_r^x = k_t^x$

wave equation - medium 1: $k_i^2 v^2 = \omega^2 = k_r^2 v^2$

$\Rightarrow k_i^z = -k_r^z$ & $\theta_i = \theta_r$

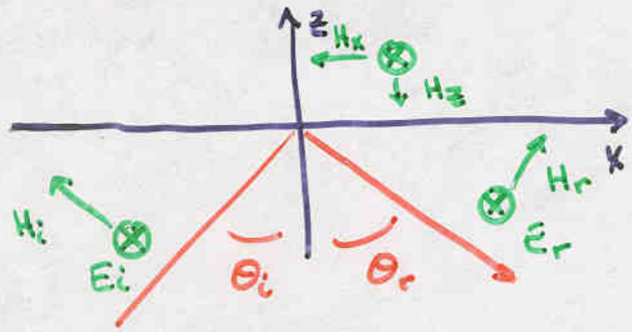
wave equation - medium 2: $(k_t^x)^2 + (k_t^z)^2 = \omega^2/v_2^2$

$$(k_t^z)^2 = \omega^2/v_2^2 - (k_t^x)^2 = \omega^2/v_2^2 - (k_i^x)^2 = \omega^2/v_2^2 - \omega^2/v_1^2 \sin^2\theta_i$$

$$= \frac{\omega^2}{c^2}\left[n_2^2 - n_1^2\sin^2\theta_i\right] \Rightarrow k_t^z = i\frac{\omega}{c}n_2\left[\frac{n_1^2}{n_2^2}\sin^2\theta_i - 1\right]^{1/2}$$

Total Internal Reflection - Field Strengths

Transverse Electric



Continuity of $E_{Transverse}$

$$E_{oi} + E_{or} = E_{ot}$$

Continuity of $H_{transverse}$

$$(H^{\bullet}_{oi} - H_{or}) \cos\theta_i = H^{x}_{ot}$$

$$\sqrt{\frac{\mu_1}{\varepsilon_1}}\,(E_{oi} - E_{or}) \cos\theta_i = H^{x}_{ot}$$

Continuity of B_{normal} or D_{normal}

$$\mu_1 \, (H^{\bullet}_{oi} + H_{or}) \sin\theta_i = \mu_2 H^{z}_{ot}$$

$\nabla \cdot B = 0$ for H^{x}_{ot} and H^{z}_{ot}

$$k^{x}_{t} H^{x}_{ot} + k^{z}_{t} H^{z}_{ot} = 0$$

Total Internal Reflection (assume $\mu_1 = \mu_2$)

Transverse Electric

$$\left(\frac{E_{or}}{E_{oi}}\right) = \frac{\cos\theta_i + i\left[\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_i - i\left[\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}$$

$$\left(\frac{E^y_{ot}}{E_{oi}}\right) = \frac{2\cos\theta_i}{\cos\theta_i - i\left[\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}$$

$$\left(\frac{H^x_{ot}}{E_{oi}}\right) = -\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2} \frac{2 i n_1 \cos\theta_i \left[\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_i - i\left[\sin^2\theta_i - \left(n_2/n_1\right)^2\right]^{1/2}}$$

$$\left(\frac{H^z_{ot}}{E_{oi}}\right) = \left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2} \frac{n_1 \sin 2\theta_i}{\cos\theta_i - i\left[\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}$$

Notes:

1. $\left|\frac{E_{or}}{E_{oi}}\right| = 1 \Rightarrow$ R = coefficient of reflection $= 1$

2. $\frac{E_{or}}{E_{oi}} = e^{i\varphi} \qquad \varphi = 2\tan^{-1}\left\{\frac{\left[\sin^2\theta_i - \frac{n_1^2}{n_2^2}\right]^{1/2}}{\cos\theta_i}\right\}$

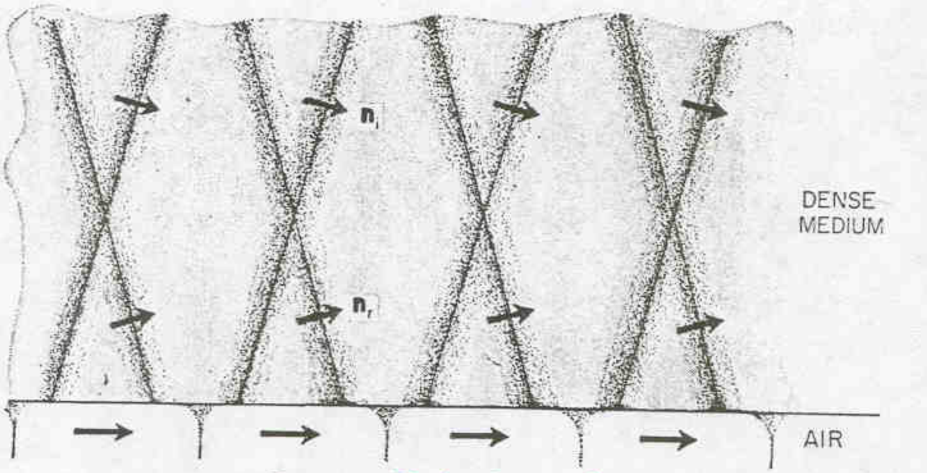
3. Energy Flow — Poynting vector.

$$S_{avg} = \frac{1}{2}\mathrm{Re}\langle E \times H^*\rangle = \frac{1}{2}\mathrm{Re}(E^y_{ot} H^{z*}_{ot})\hat{x} - \frac{1}{2}\mathrm{Re}(E^y_{ot} H^{x*}_{ot})\hat{z}$$

$E^y_{ot} H^{x*}_{ot}$ = pure imaginary $\Rightarrow$ no energy flow out.

$E^y_{ot} H^{z*}_{ot}$ = real $\Rightarrow$ energy flow $\parallel$ to surface.

Total Internal Reflection

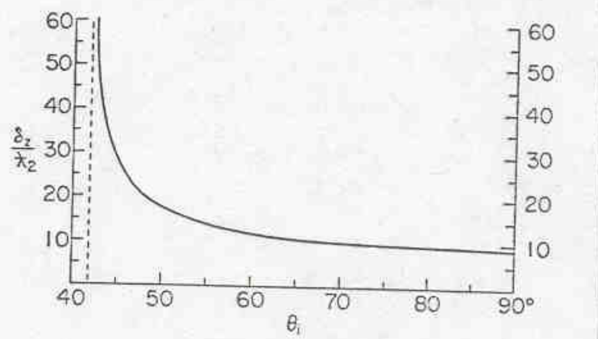


$$\vec{E}_t = \vec{E}_{t0} \left(e^{i\omega(n_1 \sin\theta_i x - t) - n_2\omega\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}} \right)$$

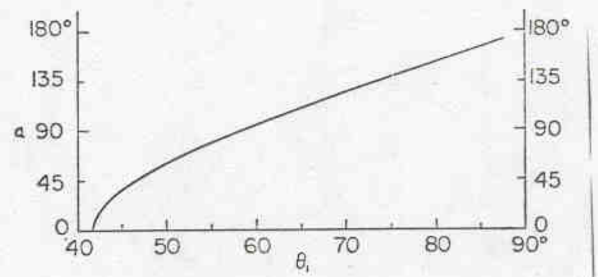
Penetration depth
beyond surface

$\lambda\!\!\!^{-} = \frac{\lambda}{2\pi}$ $\quad n_1 = 1.5$
$\quad n_2 = 1.$

λ in air.



Phase change
Reflected to Incident



Poynting vector parallel
to surface.